

## Data Analysis Formula paper:

### A. Uncertainties when doing a Measurement:

What are Measuring Instruments? The device that is used for measurement of certain physical quantity is called a measuring instrument. The measuring instruments are used for the measuring various quantities like length (e.g. Vernier caliper, micrometer), weight (e.g. digital scale, analog scale), temperature (e.g. digital thermometer), current (e.g. digital ammeter, analog ammeter), voltage (e.g. oscilloscope, digital voltmeter, analog voltmeter) etc. There are two main types of the measuring instruments: *analogue* and *digital*.

The *analogue* instruments indicate the magnitude of the quantity in the form of the pointer movement. One has to learn reading such instruments since there are certain markings on the scale. They usually indicate the values in whole numbers up to one or more decimal places depending on their smallest subdivision. The readings taken in decimal places may not always be entirely correct, since some human error is always involved in reading. The *digital* measuring instruments indicate the values of the quantity in digital format that is in numbers, which can be read easily. One doesn't need any prior training to read these instruments since they indicate the values directly in the numerical form. They can give the readings in one or more decimal places.

In your report forms, you are either asked to do the measurement once or several times along with their uncertainty, we give it a symbol  $\alpha$ :

1. Uncertainty for one measurement:

- a. Analog instrument:  $\alpha = \frac{\text{smallest subdivision}}{2}$       *answer = measurement  $\pm \alpha$*   
 b. Digital instrument:  $\alpha = \text{smallest subdivision}$       *answer = measurement  $\pm \alpha$*

2. Uncertainty for multiple measurements: (root mean square error)

$$\alpha = \frac{sx}{\sqrt{N}} \quad \text{answer} = \bar{x} \pm \alpha$$

For Casio fx-570    *stat*    *mode* > *stat* > *1-var* > (plug in your measurements) > *AC* > *shift* > *1* > *var* > *sx*

$\bar{x}$       (still in *stat mode*) *AC* > *shift* > *1* > *var* >  $\bar{x}$

For Casio (old)    *stat*    *mode* > *SD* > (plug in each value > *m+*) > *AC* > *shift* > *2* > (*S-VAR*) > *3*

$\bar{x}$       (still in *stat mode*) *AC* > *shift* > *2* > (*S-VAR*) >  $\bar{x}$

3. Uncertainties when you are asked for the value of some property which involves two or more measurements; we call this propagation of error (sometimes you might use it if it depends on one variable e.g. finding uncertainty on  $w$ ;  $w = m \cdot g$  with  $m = m \pm \alpha_m$ ):

An example of a property which involves two or more measurements is density ( $d$ ). You need to measure both the mass ( $m$ ) with an uncertainty  $\alpha_m$  and the volume ( $v$ ) with an uncertainty  $\alpha_v$  to get the density  $d = \frac{m}{v}$  with an uncertainty  $\alpha_d$ . In order to find your uncertainty you apply this formula :

$$\alpha_d = \sqrt{\left(\frac{\partial d}{\partial m} \alpha_m\right)^2 + \left(\frac{\partial d}{\partial v} \alpha_v\right)^2}$$



Where:  $\frac{\partial d}{\partial m} = \frac{1}{v}$  and  $\frac{\partial d}{\partial v} = -\frac{m}{v^2}$

Example:

Find the density of a solid of mass  $m = .80 \pm 0.10 \text{ g}$  volume  $v = 1.00 \pm 0.20 \text{ cm}^3$

$$\frac{\partial d}{\partial m} = \frac{1}{v} = 1 \text{ and } \frac{\partial d}{\partial v} = -\frac{m}{v^2} = -\frac{0.8}{1^2} = -0.8$$

$$\alpha_d = \sqrt{\left(\frac{\partial d}{\partial m} \alpha_m\right)^2 + \left(\frac{\partial d}{\partial v} \alpha_v\right)^2} = \sqrt{(1 \times 0.1)^2 + (-0.8 \times 0.2)^2} = 0.188$$

$$\text{answer: } d = \frac{m}{v} \pm \alpha_d = 0.80 \pm 0.19$$

\*note that if the quantity you are asked for depends on three measurements then under the square root for the uncertainty you would have 3 terms, and 4 terms if it depended on four measurements..

### B. Determining the number of significant figures:

When you calculate the root mean square error or you find the error by the propagation of error method you get a certain number. Take the value for the uncertainty to be the number you get up to the first significant figure and round your value e.g.

$\alpha = 0.03452$  You take  $\alpha$  to be  $\alpha = 0.03$

$\alpha = 0.76$  You take  $\alpha$  to be  $\alpha = 0.8$

$\alpha = 3.1$  You take  $\alpha$  to be  $\alpha = 3$

\*note that if your first significant figure is 1 or 2 then you take two significant figures instead of one and then round e.g.

$\alpha = 0.0143$  You take  $\alpha$  to be  $\alpha = 0.014$

$\alpha = 0.235$  You take  $\alpha$  to be  $\alpha = 0.24$

Once you get the uncertainty  $\alpha$  up to the first significant figure (or two if the first is 1 or 2) then round the value for the measurement to the same number of decimals as in the uncertainty e.g. you are measuring the density you get  $d = 0.843214$  with  $\alpha_d = 0.1463$   $\alpha_d$  becomes  $\alpha_d = 0.15$   
Then your answer for the density :  $d = 0.84 \pm 0.15$

### C. Comparison between Measured value and literature value:

You are often asked to compare your result to the literature value and find the percentage error e.g. gravity is known to be  $g_{th} = 9.81 \text{ m/s}^2$ . If you do an experiment to measure gravity experimentally and get e.g.  $g_{exp} = 9.70 \pm 0.20 \text{ m/s}^2$ , then for comparison check if the literature value belongs to the interval  $[g_{exp} - \alpha, g_{exp} + \alpha]$ .

In our example  $g_{th} = 9.8 \text{ m/s}^2$  belongs to  $[g_{exp} - \alpha, g_{exp} + \alpha] = [9.50, 9.90]$



If the theoretical value doesn't belong to the interval e.g. if  $g_{exp} = 9.50 \pm 0.20 \text{ m/s}^2$ , then  $[g_{exp}-\alpha, g_{exp}+\alpha] = [9.30, 9.70]$ , then check if the  $g_{th}$  belongs to  $[g_{exp}-2\alpha, g_{exp}+2\alpha]=[9.10, 9.90]$  and in this case it does. However, if doesn't belong to  $[g_{exp}-2\alpha, g_{exp}+2\alpha]$  your measurements is not consistent with the theoretical value; explain further what possible errors might be involved.

Sometimes, you are asked to calculate percentage error: 
$$\text{Percentage error} = \frac{|g_{exp}-g_{th}|}{g_{th}} \times 100$$

#### D. Comparison between two measurements (check if they are compatible):

If you are asked to compare the results of two experiments which measure the same thing:  
e.g.

From experiment 1 you calculated the density of oil to be  $d_1 = 0.84 \pm 0.15 \text{ g/cm}^3$ . While from experiment 2 you calculated the density of oil to be  $d_2 = 0.90 \pm 0.20 \text{ g/cm}^3$ .

Check if the two intervals  $[d_1 - \alpha_{d1}, d_1 + \alpha_{d1}]$  and  $[d_2 - \alpha_{d2}, d_2 + \alpha_{d2}]$  overlap.

In this case  $[0.69, 0.99]$  and  $[0.70, 1.10]$  overlap.

If the two intervals don't overlap, check if  $[d_1 - 2\alpha_{d1}, d_1 + 2\alpha_{d1}]$  and  $[d_2 - 2\alpha_{d2}, d_2 + 2\alpha_{d2}]$  overlap. If they don't overlap then the two experiments are not compatible.

Sometimes, you are asked to calculate percentage difference: 
$$\text{Percentage difference} = \frac{|d_1-d_2|}{\frac{d_1+d_2}{2}} \times 100$$

#### E. Linear regression:

If some quantity depends on two measurements for example finding the density for steel which requires measuring a certain steel object and its volume, we can apply the propagation for error. However, if you are asked to measure the density for steel using 3 steel objects, then in order to find the density, you can plot the 3 masses with respect to the three volumes on a graph and fit these points to a line using linear regression (again using your calculators).

Knowing that  $d = \frac{m}{v}$  then  $m = d \times v$ . If you plot m (on y axis) and v (on x axis). The slope of the line you will get is the density you are asked for. Your calculator fits the points into a line  $Y=A+BX$  with a coefficient r which determines how close your points fit this line: A for the *y-intercept*, B for the *slope*, r for the *coefficient*.

For Casio fx-570 A mode> stat> A+BX > (plug in your points and pay attention to which values you should put on your axes to get the slope you need)> AC>shift>1(stat)>REG>A  
B (still in stat mode) AC>shift>1(stat)>REG>A  
r (still in stat mode) AC>shift>1(stat)>REG>r

For Casio (old) A mode> REG> LIN > (plug in your points and pay attention to which values you should put on your axes to get the slope you need e.g. x=1 y=3 you plug it as 1,3 > M+)> AC>shift>2(S-VAR)> A  
B (still in stat mode) AC>shift>2(S-VAR)> B  
r (still in stat mode) AC>shift>2(S-VAR)> r



The relation you have sometimes might not be similar to the equation of a line e.g. you want to find the value for gravity using a ball on an inclined plane. In this case  $g \sin \theta = a$ , if you take  $a$  on y-axis and  $\theta$  on the x-axis then you would not get a line, you would get a cosine curve. Your calculator can only fit points to a line, therefore you should set  $\sin \theta$  on the x-axis. Pay attention that you have to plug in the values for  $\sin \theta$  as your values for x when using the calculator for linear regression.

Error on the slope  $\sigma_B$ : (using A,B from above)

$x_i$	$y_i$	$x^2$	$e_i = B x_i + A - y_i$	$e^2$
values	values	values	values	values
values	values	values	values	values
values	values	values	values	values
$\sum x$		$\sum x^2$		$\sum e^2$

$$\sigma_B = \sqrt{\frac{N \sum e^2}{N - 2 \Delta}}$$

Where:  $\Delta = N \sum x^2 - (\sum x)^2$

Final Answer:  $B \pm \sigma_B$

**Instructor: Razan Baltaji**  
**Email: [razan.baltaji@gmail.com](mailto:razan.baltaji@gmail.com)**  
**Office Hour: Thursday 12-1**  
**Good Luck ☺**

